

Contingent Convertible Debt and Capital Structure Decisions

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CCBs are gaining attention from both, regulators and market participants

- Contingent Convertible Bond (CCB)
 - Initially issued as debt instrument → tax deductible coupons
 - Automatically converts into equity if and when firm reaches specified level of distress
- CCBs are viewed as instruments for prudential banking regulation
 - Recent proposals: Flannery (2002, 2009), Bernanke (2009), etc.
 - Major focus on capital requirements → at the time of crisis CCB converts into equity → adequate capital ratios without additional inflow of capital
- In 2009 Lloyd's Banking Group issued \$11.6 billion of Contingent Capital (CoCo) bonds
 - Turn into equity if capital ratio falls below 5.0%
 - Yield 400 bps above traditional bonds (11.0% total)
 - Replace a portion of existing straight (regular) debt

We provide a formal, comprehensive analysis of CCBs

Questions we try to answer:

- Q1 How to value CCBs?
- Q2 Will a firm include CCBs in its capital structure if there are no regulatory conditions?
- Q3 Will a firm add CCBs to a *de novo* capital structure, given a CCB for debt constraint?
- Q4 Will a firm add CCBs to an existing capital structure, given a CBB for debt constraint?
- Q5 Can CCBs provide a useful regulatory instrument for banks too big to fail (TBTF)?
- Q6 May CCBs create an incentive for market manipulation?
- Q7 May contract restrictions maximize the regulatory benefits of CCB?
- Q8 Will CCBs magnify the incentive for assets substitution?

We use the traditional structural modeling approach based on Leland (1994)

- Debt tax advantages vs. cost of default → capital structure
- Key assumptions:
 - Firm issues equity and straight debt
 - Straight debt pays coupon c_b continually
 - Discount cash flows at constant rate r
 - Asset value follows GBM: $dA_t = \mu A_t dt + \sigma A_t dB_t^Q$
 - Tax rate $\theta \in (0, 1)$
 - Distress rate $\alpha \in [0, 1]$
- **Result 1:** Optimal default boundary $A_B = \beta(1 - \theta)c_b$
 - A_B maximizes equity value
- **Result 2:** At $\forall t$ the value of \$1 received at (hitting time) $\tau(K)$ where $K \in (A_B, A_t)$ is

$$E_t^Q [e^{-r(\tau(K)-t)}] = \left(\frac{A_t}{K}\right)^{-\gamma}$$

CCB is defined by three parameters fixed a priori: A_C , c_c , and λ

- CCB pays coupon c_c continually in time until conversion at $\tau(A_C) = \inf\{t : A_t \leq A_C\}$
 - $A_C \equiv$ conversion-triggering asset level
- c_c is tax deductible
- At conversion CCB is *fully* replaced with $(\lambda \frac{c_c}{r})$ amount of equity (valued at market price)
 - $\lambda \equiv$ conversion ratio
 - No partial conversion
 - Number of share is fixed at $\frac{\lambda c_c}{W_{tr}}$
- At conversion no inflow/outflow of capital \rightarrow no change in asset value
- A_C , c_c and λ are set when CCB is issued \rightarrow we do not solve for the optimal amount of contingent convertible debt

Condition 1: no prior default

- **Condition 1:** c_b , c_c , A_C and λ are such that the firm does not default prior to or at CCB conversion
- At conversion \rightarrow no change in the value of assets and same amount of straight debt
- After conversion \rightarrow same value maximization problem of equity holders \Rightarrow same A_B as for the case without CCB

KEY BUILDING BLOCK FOR VALUATIONS



CCB does not affect the optimal default boundary: $A_B = \beta(1 - \theta)c_b$

Closed-form solutions for values of all claims are economically intuitive

- Total value of the firm:

$$G(A_t; c_b, c_c) = A_t + \frac{\theta c_b}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) + \frac{\theta c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) - \alpha A_B \left(\frac{A_t}{A_B} \right)^{-\gamma}$$

- Equity value: $W(A_t; c_b, c_c) = A_t - \frac{c_b(1-\theta)}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) -$

$$\frac{c_c(1-\theta)}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) - A_B \left(\frac{A_t}{A_B} \right)^{-\gamma} - \left(\lambda \frac{c_c}{r} \right) \left(\frac{A_t}{A_C} \right)^{-\gamma}$$

- Value of straight debt:

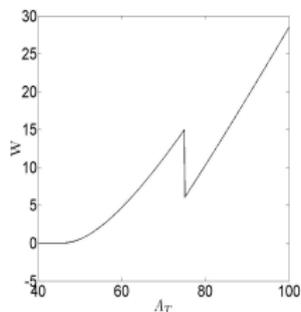
$$U(A_t; c_b, c_c) = \frac{c_b}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) + \left(\frac{A_t}{A_B} \right)^{-\gamma} (1 - \alpha) A_B$$

- Value of CCB: $U^C(A_t; c_c) = \frac{c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) + \left(\frac{A_t}{A_C} \right)^{-\gamma} \left(\lambda \frac{c_c}{r} \right)$

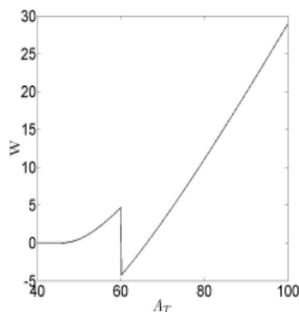
- Tax benefits: $TB(A_t; c_b, c_c) = \frac{\theta c_b}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) + \frac{\theta c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right)$

- Bankruptcy costs: $BC(A_t; c_b, c_c) = \alpha A_B \left(\frac{A_t}{A_B} \right)^{-\gamma}$

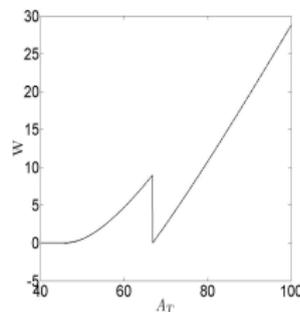
Condition 1 examples



$A_0 = \$100$, $A_C = \$75$, $\lambda = 0.9$,
 $c_c = \$0.5$, $c_b = \$5.24$, $A_B = \$46$



$A_0 = \$100$, $A_C = \$60$, $\lambda = 0.9$,
 $c_c = \$0.5$, $c_b = \$5.24$, $A_B = \$46$



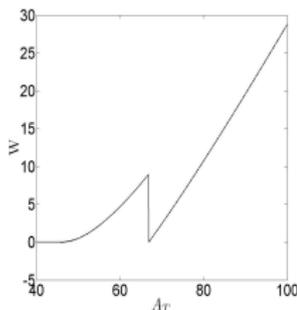
$A_0 = \$100$, $A_C = \$67$, $\lambda = 0.9$,
 $c_c = \$0.5$, $c_b = \$5.24$, $A_B = \$46$

- Based on Proposition 2: lower A_C leads to higher firm and equity value
- Define the lowest A_C that satisfies Condition 1 as

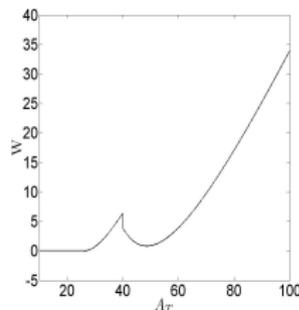
$$A_{CL} = \inf\{A_C : W(A_S; c_b, c_c) \geq 0, \forall s \geq \tau(A_C)\}$$

Condition 2: monotonicity of equity value

- **Condition 2:** c_b , c_c , A_C and λ are such that equity value ($W(A_t; c_b, c_c)$) is strictly increasing in asset level (A_t) for $A_t \geq A_C$



$A_0 = \$100$, $A_C = \$67$, $\lambda = 0.9$,
 $c_c = \$0.5$, $c_b = \$5.24$, $A_B = \$46$



$A_0 = \$100$, $A_C = \$40$, $\lambda = 0.05$,
 $c_c = \$2.5$, $c_b = \$3.0$, $A_B = \$23$

- At conversion equity holders are getting rid of the obligation to pay c_c
- High $\lambda \equiv$ expensive conversion \rightarrow equity value continues to decline
- Low $\lambda \equiv$ inexpensive conversion \rightarrow equity value increases
- Condition 2 \rightarrow alternative conversion rule based on observable equity price
- Conversion trigger: $A_C \rightarrow W_C = W(A_C; c_b, c_c)$

Small amount of CCB in the optimal capital structure

- **Assumptions about the firm**
 - No leverage
 - Issues straight debt and CCB
- Fix a sufficiently low amount of CCB that satisfies Condition 1 → find an optimal amount of straight debt that maximized firm value
- Optimal amount of straight debt (c_b^*) with CCB is the **same** as optimal **amount of straight debt** without CCB

Total firm value increases by the amount of new tax savings → original owners and equity holder of unlevered firm want to issue a CCB

- (i) Total firm value is higher by the amount of tax savings from c_c

$$G(A_t; c_b^*, c_c) = G(A_t; c_b^*, 0) + TB^C(A_t; c_b^*, c_c)$$

- (ii) Equity gets crowded by contingent convertible debt one-to-one (adjusted for new tax savings)

$$W(A_t; c_b^*, c_c) = W(A_t; c_b^*, 0) - [U^C(A_t; c_b^*, c_c) - TB^C(A_t; c_b^*, c_c)]$$

- (iii) Total tax benefits are higher by the amount of new savings

$$TB(A_t; c_b^*, c_c) = TB(A_t; c_b^*, 0) + TB^C(A_t; c_b^*, c_c)$$

- (iv) Values of straight debt and bankruptcy costs are the same

$$\begin{aligned} U(A_t; c_b^*, c_c) &= U(A_t; c_b^*, 0), \\ BC(A_t; c_b^*, c_c) &= BC(A_t; c_b^*, 0). \end{aligned}$$

Q2. Will a firm include CCBs in its capital structure if there are no regulatory conditions?

- A firm will always wish to add at least some CCB to its capital structure, to obtain the tax shield
- CCB are first added as a CCB for equity swap
 - Assets A_t are unaffected by capital changes
 - Optimal straight debt is unaffected by CCB (as long as Condition 1 holds)
- This is a losing proposition for bank regulators:
 - The default boundary A_B is unchanged
 - Fiscal deficit is expanded by new CCB tax shield
 - This may also magnify asset substitution incentive

CCB for debt swap in a *de novo* capital structure

● Assumptions about the firm

- No leverage
- Issuing straight debt and CCB

● Regulatory constraint

- Regulators constrain the total amount of debt

$$U(\bar{A}_B; \bar{c}_b, c_c) + U^C(\bar{A}_B; \bar{c}_b, c_c) = U(A_B^*; c_b^*, 0)$$

- $\bar{A}_B = \beta(1 - \theta)\bar{c}_b$; $A_B^* = \beta(1 - \theta)c_b^*$
- $U(A_B^*; c_b^*, 0) \equiv$ optimal amount of straight debt without CCB
- Firm → can choose between straight debt (no constraints) and straight debt plus CCB (regulatory constraint)

Q3. Will a firm add CCBs to a *de novo* capital structure, given a CCB for debt constraint?

- Here we impose a regulatory constraint that CCB can be added only as a swap for straight debt
- A firm will always include at least some CCB as part of a *de novo* capital structure:
 - The tax shield benefit is reduced (because CCBs convert before the straight debt defaults)
 - But the reduction in bankruptcy costs dominates
- This is perfect for prudential banking regulation:
 - Lower bankruptcy costs, lower tax shield costs
 - There is also generally less risk shifting incentive

CCB for debt swap in the existing capital structure

- **Assumptions about the firm**

- Leveraged → straight debt paying coupon \hat{c}_b ($\hat{c}_b > c_b^*$)

- **Market constraint**

- Firm wants to issue CCB and swap it for a portion of straight debt → reduce \hat{c}_b to \bar{c}_b ($\bar{c}_b < \hat{c}_b$)
- Announcement → market value of existing straight debt (still paying \hat{c}_b) rises from $U(\hat{A}_B; \hat{c}_b, 0)$ to $U(\bar{A}_B; \hat{c}_b, 0)$
- $U(\bar{A}_B; \hat{c}_b, 0)$ reflects lower default boundary due to less straight debt after swap
- **Straight debt holders must be indifferent between holding SD and swapping it for CCB**

$$U(\bar{A}_B; \bar{c}_b, c_c) + U^C(\bar{A}_B; \bar{c}_b, c_c) = U(\bar{A}_B; \hat{c}_b, 0)$$

- $\bar{A}_B = \beta(1 - \theta)\bar{c}_b$, $U(\bar{A}_B; \bar{c}_b, c_c) \equiv$ new amount of straight debt;
 $U^C(\bar{A}_B; \bar{c}_b, c_c) \equiv$ amount of CCB

Debt overhang → CCB increases total firm value but gains go to straight debt holders

- (i) For a sufficiently small amount of CCB change in total firm value is positive
- (ii) Cost of bankruptcy decreases, $BC(\bar{c}_b) < BC(\hat{c}_b)$
- (iii) Equity value decreases, $W(\bar{c}_b, c_c) - W(\hat{c}_b, 0) < 0$

Q4. Will a firm add CCBs to an existing capital structure, given a CCB for debt constraint?

- The existing equity holders will not voluntarily enter into swap of CCB for existing straight debt (given straight debt \geq optimal amount)
- While the swap will increase the firm's value (as in Q3), the gain now accrues only to the existing straight holders
 - This is debt-overhang problem
 - The problem would be reduced, even eliminated, if short-term debt could be swapped as it matured

Too-big-to-fail firms

- **Assumptions about the firm**

- 'Too-big-to-fail' (TBTF) ≡ governments take over debt at default ⇒ straight debt is risk-free
- Leveraged (straight debt paying coupon c_b) or unleveraged

- Government subsidy characteristics:

- At default worth $\frac{c_b}{r}$
- Equity holders are decision makers → maximum-equity-valuation problem does not change → default boundary A_B does not change
- Value of the subsidy at time t

$$S(A_t; c_b, 0) = \left(\frac{c_b}{r} - A_B \right) \left(\frac{A_t}{A_B} \right)^{-\gamma} = c_b \left(\frac{1}{r} - (1 - \theta)\beta \right) \left(\frac{c_b(1 - \theta)\beta}{A_t} \right)^\gamma$$

- **Increases in c_b**

Firm wants to issue as much straight debt as possible

- Total firm value increases in c_b

$$G(A_t; c_b, c_c) = A_t + \frac{\theta c_b}{r} \left(1 - \left(\frac{A_t}{A_B} \right)^{-\gamma} \right) + \frac{\theta c_c}{r} \left(1 - \left(\frac{A_t}{A_C} \right)^{-\gamma} \right) + \left(\frac{c_b}{r} - A_B \right) \left(\frac{A_t}{A_B} \right)^{-\gamma}$$

- Government sets limits on how much straight debt could be issued → fix coupon c_b^g for straight debt ⇒ **regulatory constraint**

$$U(A_t; c_b^g, 0) = U^C(A_t; \bar{c}_b, c_c) + U(A_t; \bar{c}_b, c_c)$$

$$\bar{c}_b = c_b^g - c_c \left(1 - (1 - \lambda) \left(\frac{A_t}{A_C} \right)^{-\gamma} \right)$$

- $\bar{c}_b < c_b^g \rightarrow S(A_t; c_b, 0) < S(A_t; \bar{c}_b, c_c) \Rightarrow$ **CCB reduces cost of subsidy**

Q5. Can CCBs provide a useful regulatory instrument for banks TBTF?

- Yes, a CCB for straight debt swap reduces the government subsidy by reducing the expected cost of bondholders bailouts
 - The key is to reduce the amount of straight debt
 - Taxpayers benefit from such a swap, but bank equity holders would not voluntarily participate
 - The conclusion requires Condition 1 as before
 - A mandatory swap might dominate a bank tax (by directly eliminating the bailout costs)

CCB holders might attempt to drive the equity price down → trigger conversion

- Market manipulation by CCB holders ≡ buy CCB, drive price down, trigger conversion (*get cheap equity*), sell equity when market corrects
- A_t is uncertain → A_H with p and A_L with $(1 - p)$
- Conversion based on observable equity price (as before)
- **Model driving equity price down as manipulating the market into believing that probability of A_H is p' , s.t. $p' < p$**
- Price of equity **at conversion** as the result of manipulation

$$\widetilde{W}_t = p'W(A_H; c_b, 0) + (1 - p')W(A_L; c_b, 0)$$

- Price of equity **post-conversion, after the market corrects its beliefs**

$$\widetilde{\widetilde{W}}_t = pW(A_H; c_b, 0) + (1 - p)W(A_L; c_b, 0)$$

Small λ discourages CCB holders from manipulating the equity price

- Payoff **with manipulation** (after the market corrects)

$$\Pi'_t = \lambda \frac{c_c}{r} \frac{pW(A_H; c_b, 0) - (1-p)W(A_L; c_b, 0)}{p'W(A_H; c_b, 0) - (1-p')W(A_L; c_b, 0)}$$

- Payoff **without manipulation**

$$\Pi_t = pU^C(A_H; c_b, c_c) + (1-p)\lambda \frac{c_c}{r}$$

- $\exists \lambda^* \in (0, 1)$, s.t. if $\boxed{\lambda \leq \lambda^*} \Rightarrow$ **do not manipulate** ($\Pi_t \geq \Pi'_t$), if $\lambda > \lambda^* \Rightarrow$ **manipulate** ($\Pi_t < \Pi'_t$)

- Intuition:

- Small $\lambda \equiv$ give up future c_c payments for 'too' little equity \Rightarrow do not manipulate
- Bigger $(p - p')$ (i.e., easier to manipulate) \rightarrow lower λ^*
- Bigger $(A_H - A_L)$ (i.e., bigger equity price volatility) \rightarrow lower λ^*

Equity holders might attempt to drive the equity price down → trigger conversion

- Market manipulation by equity holders ≡ buy equity, drive price down, trigger conversion (*get rid of obligation to pay c_c*), sell equity when market corrects
- **Model driving equity price down as manipulating the market into believing in poor prospects of the firm**
- Price of (old) equity **before manipulation**:

$$W(A_t; c_b, c_c)$$

- Price of (total) equity **at the point of conversion**:

$$W(A_C; c_b, 0)$$

- Price of (old) equity **post-conversion, after the market corrects**:

$$W(A_t; c_b, 0) - \lambda \frac{c_c}{r} \frac{W(A_t; c_b, 0)}{W(A_C; c_b, 0)}$$

Large λ discourages equity holders from manipulating the equity price for any realization of A_t

- **Change in value of (old) equity** the result of manipulation (post market correction)

$$\Delta W_t = W(A_t; c_b, c_c) - [W(A_t; c_b, 0) - \lambda \frac{c_c}{r} \frac{W(A_t; c_b, 0)}{W(A_C; c_b, 0)}]$$

- $\exists \lambda^{**} \in (0, 1)$, s.t. if $\lambda \geq \lambda^{**}$ \Rightarrow **do not manipulate** ($\Delta W_t \leq 0$),
if $\lambda < \lambda^{**} \Rightarrow$ **manipulate** ($\Delta W_t > 0$)
- Intuition:
 - (a) Larger $\lambda \equiv$ pay 'too' much for getting rid of c_c payments \Rightarrow do not manipulate
 - (b) Closer A_t is to A_C (i.e., easier for equity holders to manipulate) \rightarrow closer $\frac{W(A_t; c_b, 0)}{W(A_C; c_b, 0)}$ is to 1 \Rightarrow need $\lambda = 1 - \theta$ so that $\lambda \geq \lambda^{**}$ for $\forall A_t$

Q6. May CCB create an incentive for market manipulation?

- CCB may potentially create an incentive for either the CCB holders or bank equity holders to manipulate the bank's stock price to a lower value to force a CCB for equity conversion
 - CCB holders have incentive to manipulate the equity price only if the ratio of equity conversion value to CCB face value (λ) is sufficiently high to make the conversion profitable for themselves
 - Bank equity holders have incentive to manipulate the equity price only if λ is sufficiently low to make the forced conversion profitable for themselves.

Q7. May contract restrictions maximize the regulatory benefits of CCB?

- Yes, the CCB regulatory benefits generally depend on the contract and issuance terms
- Perhaps most importantly, the regulatory benefits vanish if banks simply substitute CCBs for equity
 - It is thus essential to require CCB issuance to substitute for straight debt (and not for equity)
- Also, the higher the threshold for the conversion trigger, the greater the regulatory benefits
- The conversion ratio may also determine the incentives for stock price manipulation

Table 1: Effects of CCB issuance on the capital structure of the firm

Firm	Constraint	Firm Value	Equity Holders' Value	Default Risk	Asset Substitution	Tax Savings	Other Effects	Firm Decision
Unleveraged	Sufficiently small amount of CCB	↑	↑	↔	↑	↑	n/c	Issue CCB on top of optimal amount of SD
Leveraged with SD	Sufficiently small amount of CCB	↑	↑	↔	↑	↑	n/c	Issue CCB on top of existing amount of SD
Unleveraged	Total amount of debt is fixed	↑	↑	↓	↓	~	n/c	Replace some SD with CCB
Leveraged	Total amount of debt is fixed	↑	↓	↓	↓	~	Debt overhang	Do not issue CCB
TBTF (Leveraged/Unleveraged)	Total amount of debt is fixed	↓	↓	↓	n/c	~	Reduced government subsidy	Do not issue CCB

*SD: straight debt; TBTF: Too-big-to-fail; n/c: not considered; ↑: increase; ↓: decrease; ↔: no change; ~: no effect or insignificant increase/decrease

Table 2: Incentives of CCB holders and equity holders to manipulate the stock price

Conversion Ratio	Action	Intuition
$0 < \lambda^* < \lambda$	CCB holders want to drive the stock price down to trigger conversion	If λ is high CCB holders receive a large amount of undervalued equity at conversion
$\lambda \leq \lambda^*$	CCB holders do not want to trigger conversion	If λ is low CCB holders are poorly compensated at conversion
$\lambda < 1 - \theta$	Equity holders want to drive the stock price down to trigger conversion	If λ is low equity holders can cheaply get rid of the obligation to pay c_c
$1 - \theta \leq \lambda$	Equity holders do not want to trigger conversion	If λ is high conversion is costly to equity holders

Conclusions and further research

- While CCB are highly valuable for prudential banking regulation, efficient implementation will require more detailed modeling
 - Model should allow CCB to convert in a sequence of triggers and/or the banks to commit to issue new CCBs as existing bonds convert
 - Finite maturity bonds would reduce the debt overhang costs of CCB for straight debt swaps
 - Including asset price jumps would likely improve the model's pricing accuracy
 - Finally, a full capital budgeting solution would allow the bank to buy or sell assets directly